Application of the fuzzy linear programming in the proportions of problem

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Modeling and optimization under a fuzzy environment are called fuzzy modeling and fuzzy optimization. Fuzzy multi-objective linear programming is one of the most frequently applied in fuzzy decision making techniques. Although, it has been investigated and expanded for more than decades by many researchers and from the varies point of view, it is still useful to develop new approaches in order to better fit the real world problems within framework of fuzzy multi-objective linear programming.

However, when formulating the multi-objective programming problem which closely describes and represents the real decision situation, various factors of the real system should be reflected in the description of the objective functions and the constraints. Naturally, these objective functions and the constraints involve many parameters whose possible values may be assigned by the experts. In the traditional approaches, such parameters are fixed at some values in an experimental or subjective manner through the expert's understanding of the nature of the parameters. Unfortunately, real world situations are often not deterministic. There exist various types of uncertainties in social, industrial and economic systems, such as randomness of occurrence of events, imprecision and ambiguity of system data and linguistic vagueness, etc. which come from many ways, including errors of measurement, deficiency in history and statistical data, insufficient theory, incomplete knowledge expression and the subjectivity and preference of human judgment, etc. As pointed out by Zimmermann (1978), various kinds of uncertainties can be categorized as stochastic uncertainty and fuzziness.

Stochastic uncertainty relates to the uncertainty of occurrences of phenomena or events. Its characteristics, lie in that descriptions of information are crisp and well defined; however, they vary in their frequency of occurrence. The systems with this type of uncertainty are called stochastic systems, which can be solved by stochastic optimization techniques using probability theory.

In some other situations, the decision-maker does not think about the frequently used probability distribution which is always appropriate, especially when the information is vague. It may be related to human

language and behavior, imprecise/ ambiguous system data. Such types of uncertainty are called fuzziness. It cannot be formulated and solved effectively by traditional mathematics-based optimization techniques and probability based stochastic optimization approaches.

Multi-objective Linear Programming (MOLP) Problem:

Multi-objective Linear Programming (MOLP) Problems is an interest area of research, since most real-life problems have a set of conflict objectives. A mathematical model of the MOLP problem can be written as follows:

$$\begin{aligned} &\text{Max}\, Z_1(x) = C_1 x \\ &\text{Max}\, Z_2(x) = C_2 x \\ &\text{Max}\, Z_k(x) = C_k x \\ &\text{Subject to } x \in X = \{x \in R^n / Ax = b,\ x \geq 0\} \end{aligned} \quad (1)$$

where x is an n – dimensional vector of decision variables $Z_1(x) \ldots\ldots Z_k(x)$ are k – distinct linear objective function of the decision vector . A is an mxn constraint matrix, b is an m – dimensional constant vector.

Definition 3. 1. (Complete Optimal Solution) The point $x^* \epsilon X$ is said to be a complete optimal solution of the MOLP problem (1), if $Z_i(x^*) \geq Z_i(x)$, $i = 1,2,\dots k$ for all $x \epsilon X$

In general, when the objective functions conflict with one another, a complete optimal solution may not exist and hence, a new concept of optimality, called Pareto optimality, is considered.

Definition 3. 2. (Pareto Optimal Solution) The point $x^* \in X$ is said to be a Pareto optimal solution if there does not exist $x \epsilon X$ such that if $Z_i(x) \geq Z_i(x^*)$, for all i and $Z_j(x) > Z_j(x^*)$ for at least one j

Fuzzy Multi-objective Linear Programming (FMOLP) Problem:

The model (1), all coefficients of A, b and C are crisp numbers. However, in the real-world decision

problems, a decision maker does not always know the exact values of the coefficients taking part in the problem, and that vagueness in the coefficients may not be a probabilistic type. In this situation, the decision maker can model inexactness by means of fuzzy parameter. In this section we consider a FMOLP problem with fuzzy technological coefficients and fuzzy resources. A mathematical model of the FMOLP problem can be written as follows:

$$\begin{aligned} &\text{Max} Z_1(x) = C_1 x \\ &\text{Max} Z_2(x) = C_2 x \\ &\text{Max} Z_k(x) = C_k x \\ &\text{Subject to } x \epsilon X = \{x \epsilon E^n / \tilde{A}x = \tilde{b},\ x \geq 0\} \end{aligned} \tag{2}$$

where x is an n – dimensional vector of decision variables. $Z_1(x) \dots\dots Z_k(x)$ are k - distinct linear objective

function of the decision vector x, $C_1, C_2 \dots . C_k$ are n – dimensional cost factor vectors $\tilde{A}$ is an mxn constraint fuzzy matrix $\tilde{b}$ is an m - dimensional constant fuzzy vector (fuzzy resources).

The membership function of the fuzzy matrix $\tilde{A}$ is :

$$\mu_{\tilde{A}}(x) = \begin{cases} 1, & x \leq a_{ij} \\ (a_{ij} + d_{ij} - x)/d_{ij}, & a_{ij} \leq x \leq a_{ij} + d_{ij} \\ 0, & x \geq a_{ij} + d_{ij} \end{cases}$$

where $x \epsilon R$ and $d_{ij} > 0$ (tolerance level) for $i = 1, 2, \dots, m$ and $j = 1, 2, \dots, n$.

The membership function for the fuzzy resources $\tilde{b}$ is

$$\mu_{\tilde{b}}(x) = \begin{cases} 1, & x \leq b_i \\ (b_i + p_i - x)/p_i, & b_i \leq x \leq b_i + p_i \\ 0, & x \geq b_i + p_i \end{cases}$$

where $x \epsilon R$ and $p_i > 0$ (tolerance level), for $i = 1, 2, \dots, m$.

Solution Methodology and Algorithm:

In this section, we first fuzzify the objective function in order to defuzzificate the problem (2). It is done by

calculating the lower and upper bounds of the optimal values. The bounds of the optimal values Z_q^l and Z_q^u are obtained by solving the standard linear programming problems.

$$Z_q^1 = max \sum_{j=1}^{n} c_j x_j \,, \quad q = 1,2,\dots,k$$

Subject to

$$\sum_{j=1}^{n} (a_{ij} + d_{ij}) x_j \le b_i \quad i = 1,2 \dots m \qquad (3)$$
$$x_j \ge 0$$

$$Z_q^2 = max \sum_{j=1}^{n} c_j x_j \,. \quad q = 1,2,\dots,k$$

Subject to

$$\sum_{j=1}^{n} a_{ij} x_j \le b_i + p_i, \quad i = 1,2 \dots m \qquad (4)$$
$$x_j \ge 0$$

$$Z_q^3 = max \sum_{j=1}^{n} c_j x_j \,, \quad q = 1,2,\dots,k$$

Subject to

$$\sum_{j=1}^{n} (a_{ij} + d_{ij}) x_j \le b_i + p_i \,, i = 1,2 \dots m \qquad (5)$$
$$x_j \ge 0$$

$$Z_q^4 = max \sum_{j=1}^{n} c_j x_j \,, \quad q = 1,2,\dots,k$$

Subject to

$$\sum_{j=1}^{n} a_{ij} x_j \le b_i \quad i = 1,2 \dots m \qquad (6)$$
$$x_j \ge 0$$

Let Z_q^l = min $(Z_q^1, Z_q^2, Z_q^4, Z_q^4)$ and Z_q^u=max$(Z_q^1, Z_q^2, Z_q^4, Z_q^4)$ the objective function takes the values between and while the technological coefficients take values between a_{ij} and $a_{ij} + d_{ij}$ and the right hand side numbers takes the value b_i and $a_{ij} + d_{ij}$ then the fuzzy set of j^{th} timal value, G_j which sub set for R^n is defined by

$$\mu_{G_j}(x) = \begin{cases} 0, & if\ \sum_{j=1}^{n} c_j x_j \le Z_q^l \\ (\sum_{j=1}^{n} c_j x_j - Z_q^l)/(Z_q^u - Z_q^l), & if\ Z_q^l \le \sum_{j=1}^{n} c_j x_j \le Z_q^u \\ 1, & if\ \sum_{j=1}^{n} c_j x_j \ge Z_q^u \end{cases} \qquad (7)$$

The fuzzy set of the j^{th} constraint, C_j which subset for R^n, is defined by

$$\mu_{C_j}(x) = \begin{cases} 0, & b_i \le \sum_{j=1}^{n} a_{ij} x_j \\ (b_i - \sum_{j=1}^{n} a_{ij} x_j)/(\sum_{j=1}^{n} d_{ij} x_j + p_i), & \sum_{j=1}^{n} a_{ij} x_j \le b_i \le \sum_{j=1}^{n} (a_{ij} + d_{ij}) x_j + p_i \\ 1, & b_i \ge \sum_{j=1}^{n} (a_{ij} + d_{ij}) x_j + p_i \end{cases} \qquad (8)$$

By using the definition of the fuzzy decision proposed by Bellman and Zadeh, we have:

$$\mu_D(x) = min_j(\mu_{G_j}(x), \min\ (\mu_{C_j}(x))) \qquad (9)$$

In this case the optimal fuzzy decision is a solution of the problem

$$max_{x\geq 0}(\mu_D(x)) = max_{x\geq 0}(min_j(\mu_{G_j}(x), \min(\mu_{C_j}(x)))) \quad (10)$$

Consequently, the problem (2) is reduced to the following optimization problem

$$\begin{aligned} &\max \ \lambda \\ &\lambda(Z_q^u - Z_q^l) - \sum_{j=1}^{n} c_j x_j + Z_q^l \le 0 \\ &\sum_{j=1}^{n}(a_{ij} + \lambda d_{ij})x_j + \lambda p_i - b_i \le 0 \\ &x_j \ge 0, \qquad 0 \le \lambda \le 1 \end{aligned} \quad (11)$$

Notice that, the problem (11) containing the cross product terms λx_i are not convex Therefore, the solution of the problem requires the special approach adopted for solving general non-convex optimization problems.

The Algorithm of the Fuzzy Decisive Set Method:

This method is based on the idea that, for a fixed value of λ, the problem (11) is converted in to linear

programming problem. Obtaining the optimal solution λ^* is equivalent to determining the maximum value of so that the feasible set is nonempty. The algorithm of this method for the problem (11) is presented below.

Algorithm:

Step 1:

Set λ =1 and test whether a feasible set satisfying the constraints of the problem (11) exists or not using

phase one of the simplex method. If a feasible set exists, set λ = 1 . Otherwise, set λ = u and and λ = 1 go to the next step.

Step 2:

For the value of $\lambda = \frac{\lambda^L + \lambda^R}{2}$, update the value of λ^L and λ^R using the bisection method as follows:

$\lambda^L = \lambda$, if feasible set is nonempty for λ

$\lambda^R = \lambda$, if feasible set is empty for λ

Consequently, for each λ, test whether a feasible set of the problem (11) exists or not using phase one of the Simplex method and determine the maximum value λ^* satisfying the constraints of the problem (11)

Numerical Example:

Consider the following FMOLPP

$$\begin{aligned} &\max \ Z_1(x) = 10x_1 + 11x_2 + 15x_3 \\ &\max \ Z_2(x) = 4x_1 + 5x_2 + 9x_3 \\ &\text{Subject to} \\ &\tilde{1}x_1 + \tilde{1}x_2 + \tilde{1}x_3 \le \widetilde{15} \\ &\tilde{7}x_1 + \tilde{5}x_2 + \tilde{3}x_3 \le \widetilde{80} \\ &\tilde{3}x_1 + \widetilde{4.4}x_2 + \widetilde{10}x_3 \le \widetilde{100} \\ &x_1, x_2, x_3 \ge 0 \end{aligned} \quad (12)$$

which takes fuzzy parameters as: $\tilde{1} = L(1,1)$, $\tilde{7} = L(7,4)$, $\tilde{5} = L(5,3)$, $\tilde{3} = L(3,1)$, $\widetilde{4.4} = L(4.4,2)$, $\widetilde{10} = L(10,4)$, $\widetilde{15} = L(15,5)$, $\widetilde{80} = L(80,40)$, $\widetilde{100} = L(100,30)$, as used by Shaocheng (1994).

That is $(a_{ij}) = \begin{pmatrix} 1 & 1 & 1 \\ 7 & 5 & 3 \\ 3 & 4.4 & 10 \end{pmatrix}$, $(d_{ij}) = \begin{pmatrix} 1 & 1 & 1 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{pmatrix} \Rightarrow (a_{ij} + d_{ij}) = \begin{pmatrix} 3 & 3 & 3 \\ 11 & 8 & 4 \\ 4 & 6.4 & 14 \end{pmatrix}$

$$(b_i) = \begin{pmatrix} 15 \\ 80 \\ 100 \end{pmatrix}, \; (p_i) = \begin{pmatrix} 5 \\ 40 \\ 30 \end{pmatrix} \Rightarrow (b_i + p_i) = \begin{pmatrix} 20 \\ 120 \\ 130 \end{pmatrix}$$

For defuzzification of the problem (12), we first fuzzify the objective function. This is done by calculating the lower and upper bounds of the optimal values first. The bounds of the optimal values Z_q^l and Z_q^u are obtained by solving the standard linear programming problems

$$\begin{aligned} &\max\ Z_1(x) = 10x_1 + 11x_2 + 15x_3 \\ &\max\ Z_2(x) = 4x_1 + 5x_2 + 9x_3 \\ &\text{Subject to} \\ &x_1 + x_2 + x_3 \le 15 \\ &7x_1 + 5x_2 + 3x_3 \le 80 \\ &3x_1 + 4.4x_2 + 10x_3 \le 100 \\ &x_1, x_2, x_3 \ge 0 \end{aligned} \tag{13}$$

$$\begin{aligned} &\max\ Z_1(x) = 10x_1 + 11x_2 + 15x_3 \\ &\max\ Z_2(x) = 4x_1 + 5x_2 + 9x_3 \\ &\text{Subject to} \\ &x_1 + x_2 + x_3 \le 20 \\ &7x_1 + 5x_2 + 3x_3 \le 120 \\ &3x_1 + 4.4x_2 + 10x_3 \le 130 \\ &x_1, x_2, x_3 \ge 0 \end{aligned} \tag{14}$$

$$\begin{aligned} &\max\ Z_1(x) = 10x_1 + 11x_2 + 15x_3 \\ &\max\ Z_2(x) = 4x_1 + 5x_2 + 9x_3 \\ &\text{Subject to} \\ &2x_1 + 2x_2 + 2x_3 \le 15 \\ &11x_1 + 8x_2 + 4x_3 \le 80 \\ &4x_1 + 6.4x_2 + 14x_3 \le 100 \\ &x_1, x_2, x_3 \ge 0 \end{aligned} \tag{15}$$

$$\begin{aligned} &\max\ Z_1(x) = 10x_1 + 11x_2 + 15x_3 \\ &\max\ Z_2(x) = 4x_1 + 5x_2 + 9x_3 \\ &\text{Subject to} \\ &2x_1 + 2x_2 + 2x_3 \le 20 \\ &11x_1 + 8x_2 + 4x_3 \le 120 \\ &4x_1 + 6.4x_2 + 14x_3 \le 130 \\ &x_1, x_2, x_3 \ge 0 \end{aligned} \tag{16}$$

Optimal values of these problems are $Z_1 = (189.29,\ 250,\ 110,\ 145)$ and $Z_2 = (99.29,\ 130,\ 65,\ 85)$ respectively.

Therefore, $Z_1^l = 110$, $Z_1^u = 250$, $Z_2^l = 65$ and $Z_2^l = 130$. By using these optimal values, the problem (12) can be reduced by the following non-linear programmig problem:

$$\max \ \lambda$$
$$\frac{10x_1 + 11x_2 + 15x_3 - 110}{250 - 110} \geq \lambda$$
$$\frac{4x_1 + 5x_2 + 9x_3 - 65}{130 - 65} \geq \lambda$$
$$\frac{15 - x_1 - x_2 - x_3}{2x_1 + 2x_2 + 2x_3 + 5} \geq \lambda$$
$$\frac{80 - 7x_1 - 5x_2 - 3x_3}{4x_1 + 3x_2 + x_3 + 40} \geq \lambda$$
$$\frac{100 - 3x_1 - 4.4x_2 - 10x_3}{x_1 + 2x_2 + 4x_3 + 30} \geq \lambda$$
$$x_1, x_2, x_3 \geq 0 \qquad 0 \leq \lambda \leq 1$$

that is

$$\max \ \lambda$$
$$10x_1 + 11x_2 + 15x_3 \geq 110 + 140\lambda$$
$$4x_1 + 5x_2 + 9x_3 \geq 65 + 65\lambda$$
$$(2\lambda + 1)x_1 + (2\lambda + 1)x_2 + (2\lambda + 1)x_3 \leq 15 - 15\lambda \tag{17}$$
$$(4\lambda + 7)x_1 + (3\lambda + 5)x_2 + (\lambda + 3)x_3 \leq 80 - 40\lambda$$
$$(\lambda + 3)x_1 + (2\lambda + 4.4)x_2 + (4\lambda + 10)x_3 \leq 100 - 30\lambda$$
$$x_1, x_2, x_3 \geq 0 \qquad 0 \leq \lambda \leq 1$$

Let us solve the problem (17) by using fuzzy decisive set method.

For λ = 1, the problem can be written as

$$10x_1 + 11x_2 + 15x_3 \geq 250$$
$$4x_1 + 5x_2 + 9x_3 \geq 130$$
$$3x_1 + 3x_2 + 3x_3 \leq 10$$
$$11x_1 + 8x_2 + 4x_3 \leq 40$$
$$4x_1 + 6.4x_2 + 14x_3 \leq 70$$
$$x_1, x_2, x_3 \geq 0$$

Since the feasible set is empty, by taking $\lambda^L = 0$ and $\lambda^R = 1$, the new value of $\lambda = \frac{0+1}{2} = \frac{1}{2}$ is tried

For $\lambda = \frac{1}{2} = 0.5$ the problem (17) can be written as

$$10x_1 + 11x_2 + 15x_3 \geq 180$$
$$4x_1 + 5x_2 + 9x_3 \geq 97.5$$
$$2x_1 + 2x_2 + 2x_3 \leq 12.5$$
$$9x_1 + 6.5x_2 + 3.5x_3 \leq 60$$
$$3.5x_1 + 5.4x_2 + 12x_3 \leq 85$$
$$x_1, x_2, x_3 \geq 0$$

Since the feasible set is empty, by taking $\lambda^L = 0$ and $\lambda^R = 0.5$, the new value of $\lambda = \frac{0+1/2}{2} = \frac{1}{4}$

For λ = 0.25, the problem (17) can be written as

$$10x_1 + 11x_2 + 15x_3 \geq 145$$
$$4x_1 + 5x_2 + 9x_3 \geq 81.25$$
$$1.5x_1 + 1.5x_2 + 1.5x_3 \leq 13.75$$
$$8x_1 + 5.75x_2 + 3.25x_3 \leq 70$$
$$3.25x_1 + 4.9x_2 + 11x_3 \leq 92.5$$
$$x_1, x_2, x_3 \geq 0$$

Since the feasible set is empty, by taking $\lambda^L = 0$ and $\lambda^R = 1/4$ the new value of $\lambda = \frac{0+1/4}{2} = \frac{1}{8}$ is tried

For λ = 0.125 , the problem (17) can be written as

$$10x_1 + 11x_2 + 15x_3 \geq 127.5$$
$$4x_1 + 5x_2 + 9x_3 \geq 73.125$$
$$1.25x_1 + 1.25x_2 + 1.25x_3 \leq 14.375$$
$$7.5x_1 + 5.375x_2 + 3.125x_3 \leq 75$$
$$3.125x_1 + 4.65x_2 + 10.5x_3 \leq 96.25$$
$$x_1, x_2, x_3 \geq 0$$

Since the feasible set is nonempty, by taking $\lambda^L = 1/8$ and $\lambda^R = 1/4$, the new value of $\lambda = \frac{1/8+1/4}{2} = \frac{3}{16}$ is tried

For λ = 0.19 , the problem (17) can be written as

$$10x_1 + 11x_2 + 15x_3 \geq 136.25$$
$$4x_1 + 5x_2 + 9x_3 \geq 77.19$$
$$1.38x_1 + 1.38x_2 + 1.38x_3 \leq 14.06$$
$$7.75x_1 + 5.5x_2 + 3.19x_3 \leq 72.5$$
$$3.19x_1 + 4.8x_2 + 10.75x_3 \leq 94.38$$
$$x_1, x_2, x_3 \geq 0$$

Since the feasible set is nonempty, by taking $\lambda^L = 3/16$ and $\lambda^R = 1/4$, the new value of $\lambda = \frac{3/16+1/4}{2} = \frac{7}{32}$ is tried

Similarly, we continue the above process, the following values of λ are obtained:

$$\lambda = 7/32 = 0.2188$$
$$\lambda = 13/64 = 0.2031$$
$$\lambda = 27/128 = 0.2109$$
$$\lambda = 53/256 = 0.2070$$
$$\lambda = 107/512 = 0.2089$$
$$\lambda = 213/1024 = 0.2080$$
$$\lambda = 427/2048 = 0.2085$$
$$\lambda = 853/4096 = 0.2083$$
$$\lambda = 1705/8192 = 0.2081$$
$$\lambda = 3409/16384 = 0.2081$$

Consequently, we obtain the optimal value of λ at the fifteenth iteration by using the fuzzy decisive set method. The optimal solution is $x_1 = 1.67$ $x_2 = 0$ $x_3 = 8.16$ $Z_1 = 139.1$ $Z_2 = 80.12$ and

$\lambda = 0.2081.$

In this paper, fuzzy multi-objective linear programming problem in which both the resources and the technological coefficients are fuzzy with linear membership function was studied. Further a FMLOP problem was converted into an equivalent crisp non-linear programming problem using the concept of max-min principle. The resultant non-linear programming problem was solved by fuzzy decisive set method. The discussed method was illustrated through an example. In future proposed method can be extended to solve problems like FMLOP with triangular or trapezoidal membership function and linear fuzzy fractional programming problems .

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