Comparative analysis of methods to solve the optimum routing task

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Annotation

In this article, the method to optimize the traffic with the application of a planimetric model of a network is suggested. It allows reducing volume of calculations the criterion function, it also helps to reduce number of linearly independent variables of the criterion function, that will allow to reduce time of calculation of a minimum value.

Keywords: optimum routing, planimetric model, target function, algorithms.

For the graph like set structure a network in a kind of a count. It is necessary to define all possible focused ways between each pair source-addressee. The basic mathematical model for the decision of the problem of optimum routing was offered in [1, 2]. The complexity of such problem according to [3] for the generalized algorithm of Danzig or Floyd is O $(2V^3)$. As a result of this algorithm will be found k_{mn} routes, where k_{mn} means that k routes there are exist between n and m knots. These routes must not have loops. Based on the found routes, it is possible to get both a mathematical model of streams distribution, and the target function like:

$$\sum_{ij} D_{ij}\left(F_{ij}\right), \tag{1}$$

where F_{ij} – the flow passing on a branch. This branch connects a node i to a node j;

D_{ij} – some strictly increasing function which generally determines the channel cost if the flow passes across this canal.

Flow is the amount of flows of k_{mn} of the nodes n and m passing through this branch for each couple. Thus, if we have S possible couples between sources and receivers, the quantity of variables will be equal in target function to the work S k_{mn}. As the full flow between couple of nodes n and m is usually known, one of k of flows can be expressed through other k-1 of flows. Then the total quantity linearly of independent variables will be equal in target function to S $(k_{mn}-1)$.

It is possible to offer other approach based on the tensor analysis of distributed systems of information processing. In the offered method, other approach to receiving a mathematical model of distributed systems [4] in which all flows of branches express through system of fundamental or linear and independent cycles of a graph is used. If to apply such model of the description of traffic distribution on a network, the number of operations on search of variables for target function will be reduced. It occurs because the search algorithm of fundamental cycles is based on the search algorithm in width or algorithm of creation of a spanning tree which complexity is described by the O(V) [3] function. Quantity of cycles to equally cyclomatic number of the graph r. Thus, it is necessary to mark that $r \le k_{mn}$, therefore, number of independent variables for target function will be less that will lead to faster search of a minimum of target function.

For an example, we will find optimum allocation of traffic according to a graph with the following topology, a figure 1. In branches of a graph, there is a queuing system (QS) as a mathematical model of service of information units.

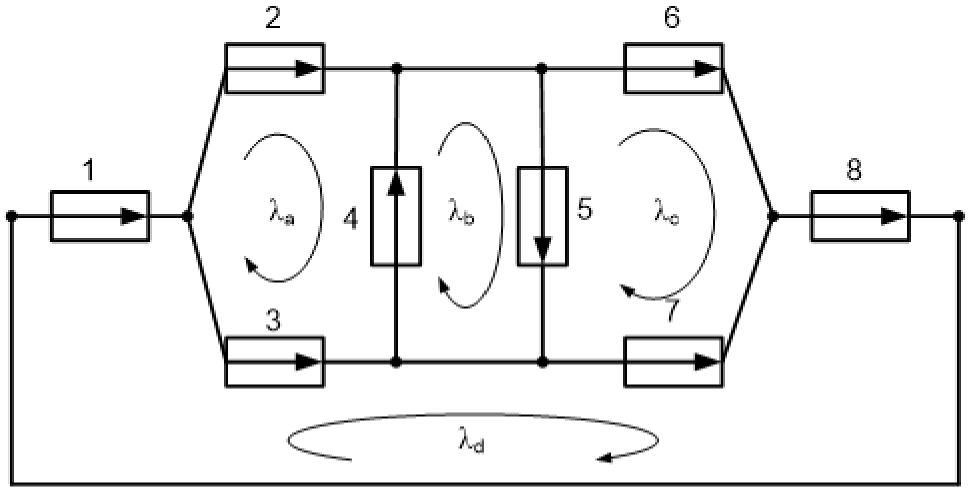

Figure 1 – The example of the researched graph

In this example, the flow is generated in a branch 1 and ends in a branch 8. Closing of a branch 1 with a branch 8 is necessary for support of saving circulation of a flow [5]. The system of linear and independent planimetric intensivnost allows defining single-digit loading in any branch through the following system of equations:

$$\begin{cases} \lambda_1 = \lambda_d \\ \lambda_2 = \lambda_a \\ \lambda_3 = -\lambda_a + \lambda_d \\ \lambda_4 = -\lambda_a + \lambda_b \\ \lambda_5 = \lambda_b - \lambda_c \\ \lambda_6 = \lambda_c \\ \lambda_7 = -\lambda_c + \lambda_d \\ \lambda_8 = \lambda_d \end{cases} \Rightarrow \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \\ \lambda_5 \\ \lambda_6 \\ \lambda_7 \\ \lambda_8 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 \\ -1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \lambda_a \\ \lambda_b \\ \lambda_c \\ \lambda_d \end{bmatrix} \Rightarrow A = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 \\ -1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad (2)$$

where $\lambda_i \; i \in [1 \ldots 8]$ - intensity in branches of a graph;

$\lambda_i \; i \in [a \ldots d]$ - planimetric intensity.

Further, we will express loadings of each branch through planimetric intensity:

$$P_{ветви} = \begin{bmatrix} \rho_1 \\ \rho_2 \\ \rho_3 \\ \rho_4 \\ \rho_5 \\ \rho_6 \\ \rho_7 \\ \rho_8 \end{bmatrix} = T \cdot \Lambda = \begin{bmatrix} t_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & t_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & t_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & t_4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & t_5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & t_6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & t_7 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & t_8 \end{bmatrix} \begin{bmatrix} \lambda_d \\ \lambda_a \\ -\lambda_a + \lambda_d \\ -\lambda_a + \lambda_b \\ \lambda_b - \lambda_c \\ \lambda_c \\ -\lambda_c + \lambda_d \\ \lambda_d \end{bmatrix} = \begin{bmatrix} \lambda_d t_1 \\ \lambda_a t_2 \\ (-\lambda_a + \lambda_d) t_3 \\ (-\lambda_a + \lambda_b) t_4 \\ (\lambda_b - \lambda_c) t_5 \\ \lambda_c t_6 \\ (-\lambda_c + \lambda_d) t_7 \\ \lambda_d t_8 \end{bmatrix}, (3)$$

where ρ_i, t_i, λ_i - loading of communication link, average duration of tinning of unit of informatior planimetric intensity;

A – the transposed matrix of linear and independent circuits.

If as QS to take M/M/1 that (1) can be written in the following look:

$$F(\rho_i) = \sum_{i=1}^{k} \frac{\rho_i}{1 - \rho_i}$$

That is, target function is equal to the amount of all average lengths of queues in eacl communication link. Restrictions for variables: $0 \le \rho_i \le 1 \quad \forall i$.

We will set the following numerical parameter values of the researched system. A holding tim of each QS, except QS 6, we will accept equal 0.05, the holding time in QS 6 is equal 0.1. Planimetri intensity of flows: λ_a=2.897, λ_b=2.897, λ_c=2.531, we will add these values in (2) and will turn ou λ_1=10, λ_2=2.897, λ_3=7.103, λ_4=0, λ_5=0.366, λ_6=2.531, λ_7=7.469, λ_8=10.

Thus, the offered method allows to define target function with smaller costs of computation and to find its minimum. Besides complexity of algorithm of receiving target function and quantity of independent variables in target function grow linearly with growth of quantity of branches, in difference from the compared algorithms where complexity of computation is described by degree dependence.

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